Appendix C

Mathematical Formulas

This appendix—by no means exhaustive—serves as a handy reference. It does contain all the formulas needed to solve circuit problems in this book.

C.1 Quadratic Formula

The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

C.2 Trigonometric Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sec x = \frac{1}{\cos x}, \qquad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}, \qquad \cot x = \frac{1}{\tan x}$$

$$\sin(x \pm 90^\circ) = \pm \cos x$$

$$\cos(x \pm 90^\circ) = \mp \sin x$$

$$\sin(x \pm 180^\circ) = -\sin x$$

$$\cos(x \pm 180^\circ) = -\cos x$$

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \text{(law of sines)}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \qquad \text{(law of cosines)}$$

$$\frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)} = \frac{a - b}{a + b} \qquad \text{(law of tangents)}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$2 \sin x \sin y = \cos(x - y) - \cos(x + y)$$

$$2 \sin x \cos y = \sin(x + y) + \sin(x - y)$$

$$2 \cos x \cos y = \cos(x + y) + \cos(x - y)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$K_1 \cos x + K_2 \sin x = \sqrt{K_1^2 + K_2^2}\cos\left(x + \tan^{-1}\frac{-K_2}{K_1}\right)$$

$$e^{jx} = \cos x + j\sin x \qquad \text{(Euler's formula)}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$1 \text{ rad} = 57.296^\circ$$

C.3 Hyperbolic Functions

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

C.4 Derivatives

If $U = U(x)$, $V = V(x)$, and $a = \text{constant}$,

$$\frac{d}{dx}(aU) = a\frac{dU}{dx}$$

$$\frac{d}{dx}(UV) = U\frac{dV}{dx} + V\frac{dU}{dx}$$

$$\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$$

$$\frac{d}{dx}(aU^n) = naU^{n-1}$$

$$\frac{d}{dx}(a^U) = a^U \ln a\frac{dU}{dx}$$

$$\frac{d}{dx}(e^U) = e^U\frac{dU}{dx}$$

$$\frac{d}{dx}(\sin U) = \cos U\frac{dU}{dx}$$

$$\frac{d}{dx}(\cos U) = -\sin U\frac{dU}{dx}$$

C.5 Indefinite Integrals

If $U = U(x)$, $V = V(x)$, and $a = \text{constant}$,

$$\int a\,dx = ax + C$$

$$\int U\,dV = UV - \int V\,dU \qquad \text{(integration by parts)}$$

$$\int U^n\,dU = \frac{U^{n+1}}{n+1} + C, \qquad n \neq 1$$

$$\int \frac{dU}{U} = \ln U + C$$

$$\int a^U\,dU = \frac{a^U}{\ln a} + C, \qquad a > 0, a \neq 1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1) + C$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2) + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax) + C$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax) + C$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax) + C$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax) + C$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C, \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \qquad a^2 \neq b^2$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{x^2\,dx}{a^2+x^2} = x - a\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{(a^2+x^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right) + C$$

C.6 Definite Integrals

If m and n are integers,

$$\int_0^{2\pi} \sin ax\,dx = 0$$

$$\int_0^{2\pi} \cos ax\,dx = 0$$

$$\int_0^{\pi} \sin^2 ax\,dx = \int_0^{\pi} \cos^2 ax\,dx = \frac{\pi}{2}$$

$$\int_0^{\pi} \sin mx \sin nx\,dx = \int_0^{\pi} \cos mx \cos nx\,dx = 0, \quad m \neq n$$

$$\int_0^{\pi} \sin mx \cos nx\,dx = \begin{cases} 0, & m+n = \text{even} \\ \dfrac{2m}{m^2-n^2}, & m+n = \text{odd} \end{cases}$$

$$\int_0^{2\pi} \sin mx \sin nx\,dx = \int_{-\pi}^{\pi} \sin mx \sin nx\,dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$\int_0^{\infty} \frac{\sin ax}{x}\,dx = \begin{cases} \dfrac{\pi}{2}, & a > 0 \\ 0, & a = 0 \\ -\dfrac{\pi}{2}, & a < 0 \end{cases}$$

C.7 L'Hopital's Rule

If $f(0) = 0 = h(0)$, then

$$\lim_{x \to 0} \frac{f(x)}{h(x)} = \lim_{x \to 0} \frac{f'(x)}{h'(x)}$$

where the prime indicates differentiation.